

Multivariate Hypergeometric

$$f(x) = \frac{\binom{N_1}{x_1} \binom{N_2}{x_2} \cdots \binom{N_k}{x_k}}{\binom{N}{n}}$$

$$N_1 + \cdots + N_k = N$$

$$x_1 + \cdots + x_k = n$$

Discrete Distributions

Bernoulli

$$0 < p < 1$$

$$f(x) = p^x (1-p)^{1-x}, \quad x = 0, 1$$

$$M(t) = 1 - p + pe^t$$

$$\mu = p, \quad \sigma^2 = p(1-p)$$

Binomial

$$b(n, p)$$

$$0 < p < 1$$

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$$M(t) = (1 - p + pe^t)^n$$

$$\mu = np, \quad \sigma^2 = np(1-p)$$

Geometric

$$0 < p < 1$$

$$f(x) = (1-p)^{x-1} p, \quad x = 1, 2, 3, \dots$$

$$M(t) = \frac{pe^t}{1 - (1-p)e^t}, \quad t < -\ln(1-p)$$

$$\mu = \frac{1}{p}, \quad \sigma^2 = \frac{1-p}{p^2} \quad \therefore P(X > k) = q^k$$

Hypergeometric

$$N_1 > 0, \quad N_2 > 0$$

$$N = N_1 + N_2$$

$$f(x) = \frac{\binom{N_1}{x} \binom{N_2}{n-x}}{\binom{N}{n}}, \quad x \leq n, x \leq N_1, n-x \leq N_2$$

$$\mu = n \left(\frac{N_1}{N} \right), \quad \sigma^2 = n \left(\frac{N_1}{N} \right) \left(\frac{N_2}{N} \right) \left(\frac{N-n}{N-1} \right)$$

Negative Binomial

$$0 < p < 1$$

$$r = 1, 2, 3, \dots$$

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \quad x = r, r+1, r+2, \dots$$

$$M(t) = \frac{(pe^t)^r}{[1 - (1-p)e^t]^r}, \quad t < -\ln(1-p)$$

$$\mu = r \left(\frac{1}{p} \right), \quad \sigma^2 = \frac{r(1-p)}{p^2}$$

Poisson

$$0 < \lambda$$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots$$

$$M(t) = e^{\lambda(e^t - 1)}$$

$$\mu = \lambda, \quad \sigma^2 = \lambda$$

Uniform

$$m > 0$$

$$f(x) = \frac{1}{m}, \quad x = 1, 2, \dots, m$$

$$\mu = \frac{m+1}{2}, \quad \sigma^2 = \frac{m^2-1}{12}$$

Multinomial

$$f(x) = \frac{n!}{x_1! x_2! \cdots x_k!} p_1^{x_1} \cdots p_k^{x_k}$$

$$x_1 + \cdots + x_k = n$$

$$p_1 + \cdots + p_k = 1$$

Cauchy: $X \sim (m, d)$

$$f(x) = \frac{1}{\pi d \left(1 + \left(\frac{x-m}{d}\right)^2\right)}$$

Continuous Distributions

Beta

$$0 < \alpha$$

$$0 < \beta$$

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha + \beta}, \quad \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta + 1)(\alpha + \beta)^2}$$

Chi-square

$$\chi^2(r)$$

$$r = 1, 2, \dots$$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1} e^{-x/2}, \quad 0 \leq x < \infty$$

$$M(t) = \frac{1}{(1-2t)^{r/2}}, \quad t < \frac{1}{2}$$

$$\mu = r, \quad \sigma^2 = 2r$$

Exponential

$$0 < \theta$$

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad 0 \leq x < \infty$$

$$M(t) = \frac{1}{1-\theta t}, \quad t < \frac{1}{\theta}$$

$$\mu = \theta, \quad \sigma^2 = \theta^2$$

Gamma

$$0 < \alpha$$

$$0 < \theta$$

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, \quad 0 \leq x < \infty$$

$$M(t) = \frac{1}{(1-\theta t)^\alpha}, \quad t < \frac{1}{\theta}$$

$$\mu = \alpha\theta, \quad \sigma^2 = \alpha\theta^2$$

Normal

$$N(\mu, \sigma^2)$$

$$-\infty < \mu < \infty$$

$$0 < \sigma$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty$$

$$M(t) = e^{\mu t + \sigma^2 t^2/2}$$

$$E(X) = \mu, \quad \text{Var}(X) = \sigma^2$$

Uniform

$$U(a, b)$$

$$-\infty < a < b < \infty$$

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

$$M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}, \quad t \neq 0; \quad M(0) = 1$$

$$\mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$$

$$\Gamma(x) \equiv \int_0^\infty x^{x-1} e^{-x} dx$$

$$\therefore \Gamma(1) = 1 \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(n) = (n-1)! \quad \text{if } n \in \mathbb{Z}^+$$